The *E*-Plane Step-Diaphragm Junction Discontinuity

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***Abstract*—A two-to-one *E*-plane step in rectangular guide is combined with a capacitive diaphragm at the junction. Singular integral equation techniques are applied, and the effect of the diaphragm in distorting the junction field is determined. The junction susceptance is shown to be augmented by a term corresponding directly to the diaphragm susceptance, with, rather surprisingly, zero net coupling effect between the step and the diaphragm.**

I. Introduction

ONE of the more complicated waveguide discontinuity geometries that lends itself to solution by the singular integral equation technique is the two-to-one *E*-plane step with a capacitive diaphragm in the unstepped wall in the plane of the junction. It happens that no additional difficulty comes from assuming different media on either side of the junction, and in [1] and [2] the analysis, in the absence of the diaphragm, the solution was completed for the quasi-static case. We examine here the changes necessary to incorporate the diaphragm, and the effect on the junction field and equivalent circuit elements.

II. The Equation for the Junction Aperture Field

The arrangement is shown in Fig. 1, and following the method of [2] we express the electromagnetic fields in the guide in terms of Fourier integrals of the unknown junction field $E(\pi y/b)$. Using a well known theorem [2, sec. 1.4.2] the geometry actually investigated is a parallel-plate geometry. The transition to rectangular waveguide is achieved by replacing λ by

$$\lambda(1-\lambda^2/4a^2)^{-1/2}$$

in the final equations.

By requiring the expressions for the magnetic fields on either side of the junction to be equal at the junction, an integral equation for the aperture field is obtained. It is identical to [2, eq. (8.49)] except that the range of integration, and the range of validity of the equation, is reduced because of the reduced aperture due to the presence of the diaphragm. The altered equation takes the form

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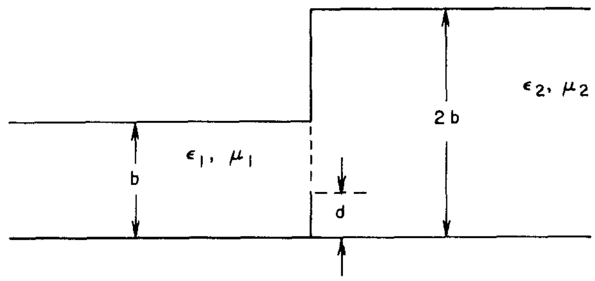

Fig. 1. Waveguide step with aperture diaphragm.

$$-(1-R)/\hat{\zeta}_1+(1+R)/2\hat{\zeta}_2$$
$$=-j\omega\epsilon_1\frac{2}{\pi}\int_{\pi d/b}^{\pi}E(\phi)\sum_1^{\infty}\frac{\cos n\theta\cos n\phi}{\Gamma_{1,n}}\,d\phi$$
$$-j\omega\epsilon_2\frac{1}{\pi}\int_{\pi d/b}^{\pi}E(\phi)\sum_1^{\infty}\frac{\cos\frac{1}{2}n\theta\cos\frac{1}{2}n\phi}{\Gamma_{2,n}}\,d\phi \quad (1)$$

where R is the dominant mode reflection coefficient into incident guide

$$\hat{\zeta}_{1,2}=(\mu_{1,2}/\epsilon_{1,2})^{1/2}$$

$$\Gamma_{1,n}=\left[(n\pi/b)^2-(2\pi/\lambda_0)^2(\epsilon_1\mu_1/\epsilon_0\mu_0)\right]^{1/2}\sim n\pi/b, \quad \text{for large } n$$

$$\Gamma_{2,n}=\left[(n\pi/2b)^2-(2\pi/\lambda_0)^2(\epsilon_2\mu_2/\epsilon_0\mu_0)\right]^{1/2}\sim n\pi/2b, \quad \text{for large } n$$

b is the incident guide height, d is the diaphragm insert

$$\theta,\phi=(\pi y/b),(\pi y'/b)$$

and λ_0 is the free-space wavelength. Note that all modes except the $n=0$ mode are normally nonprogating.

Proceeding to the quasi-static limit, and differentiating with respect to θ leads, as in [2], to the following singular integral equation for $E(\phi)$

$$\int_{\pi d/b}^{\pi}\frac{E(\phi)}{\sin^2\left(\frac{1}{2}\phi\right)-\sin^2\left(\frac{1}{2}\theta\right)}\left[\cos\tfrac{1}{2}\phi+\alpha^2\cos\tfrac{1}{2}\theta\right]d\phi=0, \quad \pi d/b<\theta<\pi \quad (2)$$

where

$$\alpha^2=1+2\epsilon_1/\epsilon_2=3, \quad \text{for equal media}$$

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and let

$$\beta=(1/\pi)\tan^{-1}\alpha=1/3, \qquad \text{for equal media.} \qquad (3)$$

III. Solution of the Singular Integral Equation

In (2) we make the substitutions

$$u=\sin\tfrac{1}{2}\theta$$
$$v=\sin\tfrac{1}{2}\phi$$
$$c=\cos(\pi d/2b)$$
$$s=\sin(\pi d/2b)$$
$$E(\phi)=F(v)dv/d\phi$$

giving

$$\int_s^1 \frac{F(v)}{u^2-v^2}\left[(1-v^2)^{1/2}+\alpha^2(1-u^2)^{1/2}\right]dv=0, \qquad s<u<1. \qquad (4)$$

The key variable change needed to put this into standard form is

$$(1-u^2)^{1/2}=c(1-\xi^2)^{1/2}$$
$$(1-v^2)^{1/2}=c(1-\eta^2)^{1/2}$$
$$F(v)=G(\eta)d\eta/dv. \qquad (5)$$

Then (4) becomes

$$\int_0^1 \frac{G(\eta)}{\xi^2-\eta^2}\left[(1-\eta^2)^{1/2}+\alpha^2(1-\xi^2)^{1/2}\right]d\eta=0, \qquad 0<\xi<1. \qquad (6)$$

This is solved as in [2] to give

$$G(\eta)=(1+R)\sin\beta\pi\left[\frac{H^\beta+H^{1-\beta}}{1-\eta}\right] \qquad (7)$$

with

$$H=(1-\eta)/(1+\eta).$$

Returning to the original variables, and putting R in terms of the normalized input susceptance of the junction B gives

$$E\left(\frac{\pi y}{b}\right)=\frac{\sin\beta\pi\sin(\pi y/b)}{2(1+jB+\hat{\xi}_1/2\hat{\xi}_2)\left[\sin^2(\pi y/2b)-s^2\right]^{1/2}} \cdot\left[\frac{H^\beta+H^{1-\beta}}{c-\left[\sin^2(\pi y/2b)-s^2\right]^{1/2}}\right] \qquad (8)$$

with

$$H=\frac{c-\left[\sin^2(\pi y/2b)-s^2\right]^{1/2}}{c+\left[\sin^2(\pi y/2b)-s^2\right]^{1/2}}.$$

In the absence of the diaphragm, $s=0$, $c=1$, and $H=\tan^2[(\pi/4)(1-y/b)]$. Clearly, the field in (8) is distorted by the presence of the diaphragm, though its affinity to the undistorted field is apparent.

IV. The Junction Susceptance

Equation (2) was derived from (1) by using the quasi-static forms $\Gamma_{1,n}$ and $\Gamma_{2,n}$ and differentiating with respect to θ. The differentiation was necessary in order to handle the equation with available techniques, but in the process information was lost, namely, the constant terms in (1). Accordingly, it is necessary, as in [2], to return to the undifferentiated form to determine the remaining unknown B in (8). The method follows closely the details in the reference; the only difference comes from the variable change of (5). Since a logarithm is involved, $\log(1-v^2)^{1/2}=\log c+\log(1-\eta^2)^{1/2}$ and an extra additive term in $\log c$ enters. This is the *only* change, and as a result the normalized junction susceptance becomes, after some manipulations,

$$B=\frac{2b}{\lambda}\left(2+\frac{\epsilon_2}{\epsilon_1}\right)\left[\tfrac{1}{2}\pi\cot\beta\pi-2\log 2 -\gamma-\Psi(1-\beta)-\log c\right] \qquad (9)$$

where

$$\lambda=\lambda_0(\epsilon_0/\epsilon_1)^{1/2}$$

γ is the Euler's constant $=0.5772$, and Ψ is the logarithmic derivative of the gamma function. The initial terms in β are the same as for a junction with no diaphragm. For equal media the expression in square brackets simplifies to

$$(3/2)\log 3-2\log 2-\log c.$$

Now, from [2, eq. (6.69)] (with s in that equation equal to c^2 as used here), the susceptance of a diaphragm of insertion d into an unstepped guide of height b is B_0 where $B_0=-(8b/\lambda)\log c$. If we consider the diaphragm in Fig. 1 to be half in a guide of height b and half in a guide of height $2b$ we might expect to get, on a rather naïve picture of two noninteracting capacitors in parallel, a value

$$B_0'=-\frac{4b}{\lambda}\log c-\frac{4(2b)}{\lambda}\log c'$$

where $c'=\cos(\pi d/4b)$. Very approximately, for small d, we can write

$$c'\approx 1-\tfrac{1}{2}(\pi d/4b)^2=1-\tfrac{1}{2}(\pi d/2b)^2/4$$
$$\approx\left[1-\tfrac{1}{2}(\pi d/2b)^2\right]^{1/4}\approx c^{1/4}$$

when $B_0'\approx-(2b/\lambda)(2+1)\log c$. This would be the case for equal media. Since the capacitances are proportional to the permittivities, the term $(2+1)$ would be replaced by $(2\epsilon_1+\epsilon_2)/\epsilon_1$, leading to the general form of the additive term in $\log c$ in (9), which is, therefore, understandable from this point of view. But what is truly remarkable about this formula is that *the rest of the equation is unaffected* by the presence of the diaphragm. This means that although the junction field is distorted by the insertion of the diaphragm the *net* coupling between the diaphragm and the junction is zero. This result was quite unexpected. If the formula for a double diaphragm, one from each side in an unstepped waveguide, is examined, it is readily seen that the separate capacitance terms do not simply add. There is a mutual coupling between them, as would indeed be expected, accompanying the change of aperture field that one diaphragm induces on the other. Apparently, this rather general feature is absent in the two-to-one waveguide step with a junction diaphragm. Although the field is distorted by the diaphragm, the net excess charge due to the step is merely redistributed, a rather unexpected outcome. The formula has been checked by colleagues who find it correct, but who have no physical explanation for the finding. It is not known if it is a freak result coincidental on the two-to-one step ratio. Since, currently, the singular integral equation technique can only handle this case the effect of altering the step ratio on the diaphragm interaction is not, at the present time, resolvable.

Acknowledgment

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The Susceptance of an Annular Metallic Strip in a Circular Waveguide with Incident TE_{01} Mode

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***Abstract*—The principal aims of this paper are twofold: 1) to solve the problem of the scattering of a thin, perfectly conducting annular strip suspended in a multimodal circular waveguide in which any number of TE_{0n} modes can propagate, and with the aid of this result, 2) to give the susceptance of the thin annular strip in monomodal circular guide with an incident TE_{01} mode. These are treated with a variational approach.**

Applying the appropriate Green's functions to the continuity equations for the transverse electric field yields a variational expansion for the scattering matrix elements. This is treated with a Rayleigh–Ritz procedure and matrix methods.

Curves of normalized susceptance as a function of the free-space wavelength and the size of the annular metallic strip are shown. These results are in good agreement with experimental data.

Tables of the scattering coefficients for a typical wavelength versus strip size are also included.

I. Introduction

IN RECENT YEARS, an experimental millimeter-wave telecommunication system has been constructed in our country [1]. The circular waveguide capable of propagating the dominant circular-electric mode is ideally suited as a low-loss transmission line in the millimeter-wave region. Inductive metal irises of zero thickness in such circular waveguides (see Fig. 1) have been investigated and documented [2]–[4]. In these reports, the finding of the susceptance when the TE_{01} mode was incident was the main goal.

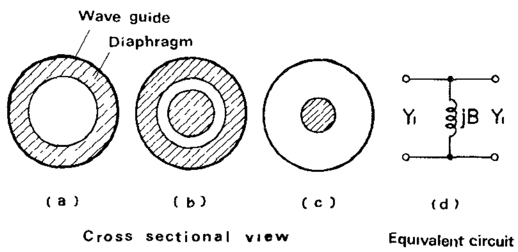

Fig. 1. Metallic plate diaphragms of zero thickness and their equivalent circuit (TE_{01} mode in circular waveguide).

Problems of susceptance for discontinuities in waveguides have been widely studied during the past decade. Except for a few special discontinuities, exact solutions are not available and approximate methods must be used. Of the approximate techniques, the variational and integral-equation methods are applicable to a wide range of problems, and produce sufficiently accurate results for most purposes. The former method is described by Collin

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